The Production Function - Recent calculation and analysis of the world data

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Abstract. The macroeconomics studies understands the economy as a mathematical form that reflects reality, therefore the base of this field finds itself in equations and functions with variables that correspond to our world, in this perspective each variable changes for each country in each moment of time. Therefore there is always the need to give those calculations and results for the academic community for recent analysis, since the economy is always changing. In this understanding, the University of Pennsylvania is one of the many institutions that are responsible for providing the information collected from many countries from 1950 to 2019, and from those information I calculated the variables of the Cobb Douglas function by the development and growth accounting method, following the David Weil steps described in his books, and after that I made an statistical analysis from those results for the recent data. Through that method I found out the new outcome, physical capital, workers, and productivity to each country in 2019, and also for past periods of time, however only the 2019 results are relevant for this project. Then, after that I found out that the productivity growth rate of a country is responsible for explaining 80% of the product growth rate variation, much more than used to in 2009, in Weil studies. This shows the importance of making those calculations for the policymakers, for example, that must invest in increasing the productivity of the economy instead of the factors of production to achieve long prosperity.

Keywords. Development accounting, growth accounting, productivity, factors of production

1. Introduction

At the macroeconomics studies, there is the production function, that it is responsible for showing how the production activity works and depends on, this function can have different forms, however the most common one is the Cobb-Douglas, and it is described as:

$$Y = A \times K^{\alpha} \, L^{1-\alpha} \text{ (1)}$$

This function represents the outcome of the economy as Y, that is the product of the activity, the physical capital as K, that is literally the structural resources needed for a production, the workers as L, also it may be good to remind that K and L represents together the factors of production, and A it is the productivity, that represents the way that the factors of production mix themselves to produce product (Y).

Those variables represent the reality of a country, and each country has their own production function that varies through time. Because of that there is the difficulty in measuring those variables, also there is a lot of information needed to measure and calculate them.

In this perspective there are many institutions that collect some of this information and concentrate them in a world bank of data, the aim of this project is to calculate those variables from one of those bank of data for the past and especially for the most recent period of time, and in sequence apply statistical analyses that interpret some of the way that the production function of a certain country is working in that period in especic.

This kind of project has already been done, however the unknown information is for the recent period of the economy, that it is the most important to the studies that try to understand and predict our future.

2. Methodology

2.1 General overview

The aim of the research is to quantify and collect the most recent data for the biggest number of countries possible, where we can organize them in order by their capacity to produce. Therefore, it will need two different approaches, the development and the growth accounting.

2.2 Development accounting

The first measure of the methodology is to quantify the real numbers of the economic variables of each country. It is important to say that the countries introduced in these accounts are the ones who provided the information needed to the University of Pennsylvania, in the United States, therefore there are a group of countries that won't participate in the results. Consequently, the university created the PenWorldTable (PWT), a giant economic database last updated in 2021 [1].
Certainly the PWT have a lot of information, however it doesn't have all the calculations needed to understand the macroeconomics, so I started doing the development accounting, just like Weil described in his own book, with the objective of quantify a certain number that represent productivity, physical capital, work, technology and efficiency [2].
All of these numbers will be reflecting a level, and calculated for each country from 1950 to 2019, such as the information that the PWT can provide us.

2.3 Growth accounting

The second approche of the methodology goes by the calculations of those same variables calculated in the first approche, however now calculated in a growth rate, using 40 years of analysis because according to Weil, it is the time needed to comprehend an economy [3].

After that, I separated the countries by five groups according to their gross domestic product (GDP), that is by their economic power. Then I calculated the average of productivity and the factors of production for those groups.

Finally using a statistics analysis, I calculated the variance and covariance of the productivity and the factors of production of 2019 of the countries, and found which one of them explains better the economic growth.

It's also important to remember that I did this same analysis for the development accounting to find out which one can explain better the variation of the GPD.

Therefore, this project has the aim to repeat Weil's steps, however with more recent data and results.

3. Results

The presentation of the results can be separated by the two different approaches due to the two different calculations.

3.1 Results of the variables in level

The results I was able to obtain by the development accounting was for 150 countries, calculating the GDP in 2019 and separating them into five different groups.

I found Brazil in the middle group, with the average of the GDP of 38,240.04 and the average of the GDP relative to the United States GDP was 0.29..

The analysis of the groups extended for the calculation of the average productivity and the average of the factors of production of each groups, and the results can be observed in the table below:

Tab. 1 - Productivity average of the groups in 2019, relative to the productivity of the United States (Ai/Ausa).

poorest group	second poorest group	group of the middle	second richest group	richest group
0.22	0.37	0.53	0.63	0.88

Tab. 2 - Factors of production average of the groups in 2019, relative to the factors of the United States (Fi/Fusa).

poorest group	second poorest group	group of the middle	second richest group	richest group
0.22	0.42	0.58	0.79	0.98

As we can see on the tables, the average of the numbers are very similar between the productivity and the factors of production.

Therefore, after doing the statistical analysis, I discovered that both of the sides are important to explain the variation of the product (GPD), and almost explain at the same percentage.

However, the results show that productivity is responsible for explaining 50.34% of the variation fraction of the product in 2019, and the factors of production are responsible for explaining 49.66% of the variation fraction of the product.

This same analysis was made by David Weil in 2009, and the percentage was 53% explained by productivity and 47% explained by the factors of production [4].

3.2 Results of the variables in rate

For the analysis of economic growth the results I was able to find were for 89 countries in 2019, that

Brazil and the United States were classified at the same group, as the middle one, with an average of the growth rate of the product for 40 years (1979-2019) of 1.67% for year.

The percentages of product growth of the others groups can be observed at the table below:

Tab. 3 - Product growth rate average to 2019

poorest group	second poorest group	group of the middle	second richest group	richest group
-0.76%	1.16%	1.67%	2.18%	3.72%

After calculating product growth rate for each country and diving them in five groups, and also calculating the average of that rate for each group, I was able to calculate the average of the productivity growth rate and the factors of production growth rate for those same groups, and the results can be noticed at the tables below:

Tab. 4 - Productivity growth rate in 2019

poorest group	second poorest group	group of the middle	second richest group	richest group
-1.38%	0.31%	0.66%	1.08%	1.60%

Tab. 5 - Factors of production growth rate in 2019

poorest group	second poorest group	group of the middle	second richest group	richest group
0.58%	0.87%	1.00%	1.08%	2.16%

It is visible on the tables that the variation of the percentage points on table 5 it's bigger than on table 4, therefore the statistics analysis can corroborate with this perspective.

After calculating the variance for the product growth rate, for the factors of production growth rate and for the productivity growth rate, I calculated the covariance between the productivity and the factors of production.

Then finally, I was able to put those results into the equation theorized by Weil, and I found out that the productivity growth rate explains 80% of the variation fraction of the product growth rate, and the factors of production growth rate explains 20% of the variation fraction of the product growth rate in 2019.

Those analyses in 2009 by Weil were 68% explained by productivity growth rate and 32% explained by factors of production growth rate. [5]

4. Discussion

Those results are interesting because they are more recent, so they are more synchronized to our reality than the older results, so that we can make assumptions to explain the reasons why those numbers are higher or lower for each country and compare them to their economic policies.

Therefore the aim of this project is not to find reasons to explain how each country achieved those numbers, but only to calculate the recent data.

However, I found out that the productivity growth rate explains much more the product growth rate than used to in 2009, and by that result we can definitely confirm that it is more important to invest in the productivity than in the factors of production.

That means that to get a higher product (GDP) the economic policies should focus on getting productivity higher. It is important to remind that productivity can be separated into two variables that multiples themselves, one is technology, which is basically the machinery, and the other is efficiency, and knowing that most of the countries has technologies similar, due to the globalization, most of the differences in the productivity it's caused by the differences in the efficiency of the countries.

This variable is related with many other variables, but I can cite reaserch and corruption combate as two good examples of measures that are very important to increase the productivity of a country.

5. Conclusion

In conclusion the aim of this project was to give recent data and information about the economic variables as Weil did in 2009.

In this perspective, I have found the new numbers for the variables for each country in 2019, applying the development and the growth accounting concept.

Therefore I was able to find that in the development accounting productivity and factors of productions almost explain in the same matter the variation of the product, to sum up the investment in productivity or in factors of production will have the same effect for an analysis at one period of the time.

However, through an analysis across many periods, more specific, through 40 years, the productivity has a bigger effect over the product when we consider the growth rate.

Therefore, putting efforts to increase the productivity in a country is much more effective than putting effort on the factors of production.

However, one of the questions that has not being answered, that may interest to future's researchers it is the reason why productivity has become more important than it was in 2009.

To sum up, the fruits of this work implies in many others, due to the characteristic of being a project of giving actual and recent data to the academic community to work with and create more from it.

6. Acknowledgement

7. References

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