Averaged space-state modeling and simulation of a DC-DC Sepic-zeta converter

Klaus Malone Martins Damasceno [a]

[a] Graduate program in Electrical Engineering , Universidade Federal de São João Del Rei, São João Del Rei, Brazil, klausmmartins@gmail.com, klausmmartins@aluno.ufsj.edu.br

Abstract. The transfer function describing a power converter is a fundamental tool throughout the design and control of converters in closed loop, as seen in the voltage and average current control techniques. Therefore, this paper aims at presenting the small signal averaged space-state modeling of a DC-DC SEPIC-Zeta converter in continuous conduction mode. The resulting model can provide responses on par with precise computer simulations, making it possible to be used for designing controllers. A mathematical analysis is shown, upon which it is possible to obtain simulation results that validate the theoretical assumptions. .

Keywords. Integrated converters, Control systems, non-isolated DC-DC converters.

1. Introduction

DC-DC converters play a crucial role in adapting voltage and current levels directly between source and load, without the need of inversion and rectification through cascated converters [2]. Although being able to raise and decrease voltage levels using transformers, DC-DC converters show their potential when those are not being used. By using switching frequencies in the range of kHz, magnetics components become much smaller than in 60 Hz, resulting in a higher power density [3], efficiency, and smaller volume.

The small signal modeling will be done using the averaged space-state modeling technique proposed by Middlebrook and Ćuk in [6]. Although this technique requires a considerable amount of symbolic mathematical analysis, specially because the converter generates an eighth order sistem, the technique is a widely known and accepted across power electronics literature, justifying its use in this paper.

2. Methodology

A great number of converter modeling techniques are presented in literature. Current injection modeling consists in analysing the current contribution from each non-linear stages to the linear one, applying disturbances and determining the contribution of each non-linear stage in the output voltage [6]. In the averaged circuit method, the non-linear part is replaced by an model describing the averaged values, simulating its behaviour in low frequencies. In this model, the averaged voltages and currents through the terminals are identical to the original circuit [7]. A method very much known and cemented in literature is the averaged space-state model, which consists in modeling the system in its state-space throught the voltages and current derivatives in the capacitors and inductors, respectively.

Henceforth, disturbances are applied in the duty cycle, input voltage, output voltage, and in the state vector, resulting in a non-linear equation that must be linearized around the averaged values [4].

2.1 The SEPIC-Zeta converter

The SEPIC-Zeta converter was introduced in [5], being a cascated connetion between a SEPIC converter and a Zeta converter, using a technique presented in [1], called graft-scheme, that allows the designer to reduce the number of switches by replacing them with diodes, as long as they share a common node. The converter qualitative and quantitative analysis was throughly made in [5]. The circuit is shown in **Fig. 1.**

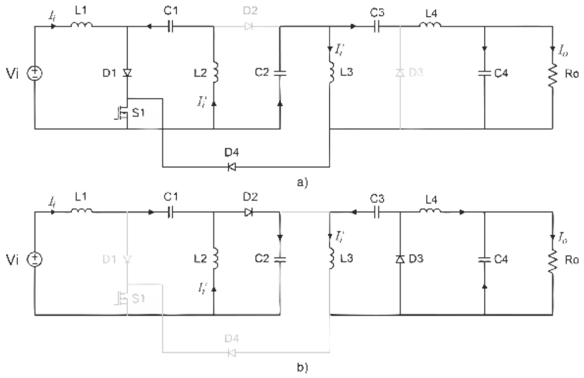

Fig. 1. SEPIC-Zeta converter in CCM. (a) First stage and (b) second stage.

2.2 Space-states

The space-state description is the standard form to describe differential equations that describe a system. Starting with a linear system, the derivatives of the state variables are expressed as linear combinations of the independent inputs of the system and of its own state variables. The state variables are usually associated to energy storage elements, and for a typical converter, the variables represent the currents in the inductors and voltages in the capacitors [8].

In any given point in time, the values of the state variables are depedent of past values, instead of the present values in the system inputs. To solve the differential equations, the initial values of the state variables must be specified. Therefore, if a state of the system is known, that is, the values of all the state variables any given time t_o, as well as the inputs, we may solve the state equations of the system to find the waveforms regarding any future time [8].

The state equations can be written in matrix form as the linear system (1.1).

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) \\ \dot{y} = Cx(t) + Du(t) \end{cases} \quad (1.1)$$

The state vector *x(t)* contain all the state variables, mainly the inductor's currents and capacitor's voltages. The input vector *u(t)* is composed of all the independent inputs of the system, with the input voltage v(t) being an example.

Middlebrook and Ćuk derived the averaged space-state equation in their famous paper in 1976 [4]. The equation, which represents the amout of disturbance in the output voltage due to a disturbance in the duty cycle is shown in (1.2).

$$Tp(s) = \frac{v_o(s)}{d'(s)} = C \cdot [s \cdot I - A]^{-1}[(A_1 - A_2) \cdot X + (B_1 - B_2 \cdot V_i)] \cdot X + (C_1 - C_2) \cdot X$$

(1.2)

The matrices *A, A_1, A_2,* are composed of constants related to the capacitances, impedances and resistances of the circuit. *B, B_1* and *B_2* represent the influence of the inputs on the derivatives of the state variables. The *y(t)* vector is called the output vector. It is normally composed just of the *x(t)* vector, that together with the *C, C_1 and C_2* matrices will express the output as a linear combination of the input variables. The *D* matrix represents a direct connection between the input and the output, which is something that will not be adressed here. The *X* vector is composed of the averaged values of the state variables. In the present case, it will be composed of equations derived in [5].

2.3 SEPIC-Zeta space-state equations

By describing the system in its standard form shown in equation (1.1), the idea of expressing the voltages on the inductors and current in the capacitors becomes clear. Starting at the first stage, from *t = 0* to *t = DTs* with *D* being the duty cycle, and *Ts* the total period of the first and second stages.

By analysing the system in **Fig. 1. (a)** using Kirchoff's laws, we arrive at the linear system presented in (1.3).

$$\begin{cases} \dot{I_{L_1}} - \dfrac{V_i}{L_1} = 0 \\ \dot{I_{L_2}} - \dfrac{V_{C_1}}{L_2} = 0 \\ \dot{I_{L_3}} - \dfrac{V_{C_2}}{L_3} = 0 \\ \dot{I_{L_4}} - \dfrac{V_{C_3} + V_{C_2} - V_{C_4}}{L_4} = 0 \\ \dot{V_{C_1}} + \dfrac{I_{L_2}}{C_1} = 0 \\ \dot{V_{C_2}} + \dfrac{I_{L_3} + I_{L_4}}{C_2} = 0 \\ \dot{V_{C_3}} + \dfrac{I_{L_4}}{C_3} = 0 \\ \dot{V_{C_4}} + \dfrac{-V_{C_4} + R_o I_{L_4}}{C_4 R_o} = 0 \end{cases}$$

(1.3)

Solving the linear system yields 8 equations that express the derivatives of the currents on the inductors and the voltages on the capacitors. The matrix form of the system, in accordance with (1.2) is shown in (1.4) and (1.5).

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{L_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{L_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{L_4} & \frac{1}{L_4} & \frac{-1}{L_4} \\ 0 & \frac{-1}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{C_2} & \frac{-1}{C_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{C_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_4} & 0 & 0 & 0 & \frac{-1}{C_4 R_o} \end{bmatrix}$$

(1.4)

$$B_1 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1.5)

Following the same steps, we repeat the process for the second stage, **Fig. 1. (b)**.

$$\begin{cases} \dot{I_{L_1}} - \dfrac{V_i - V_{C_1} - V_{C_2}}{L_1} = 0 \\ \dot{I_{L_2}} - \dfrac{-V_{C_2}}{L_2} = 0 \\ \dot{I_{L_3}} - \dfrac{-V_{C_3}}{L_3} = 0 \\ \dot{I_{L_4}} - \dfrac{-V_{C_4}}{L_4} = 0 \\ \dot{V_{C_1}} - \dfrac{I_{L_1}}{C_1} = 0 \\ \dot{V_{C_2}} - \dfrac{I_{L_2} + I_{L_1}}{C_2} = 0 \\ \dot{V_{C_3}} - \dfrac{I_{L_3}}{C_3} = 0 \\ \dot{V_{C_4}} - \dfrac{-V_{C_4} + R_o I_{L_4}}{C_4 R_o} = 0 \end{cases}$$

(1.6)

Again, solving the linear system yields 8 equations that express the derivatives of the currents on the inductors and the voltages on the capacitors. The matrix form of the system, in accordance with (1.2) is shown in (1.7) and (1.8).

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{-1}{L_1} & \frac{-1}{L_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_4} \\ \frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{C_2} & \frac{1}{C_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_4} & 0 & 0 & 0 & \frac{-1}{C_4 R_o} \end{bmatrix}$$

(1.7)

$$B_2 = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1.8)

The matrices B_1 and B_2 are the same because the input is solely the voltage source, that suffers no change during the stages.

The vector *X* is shown in (1.9).

$$X = \begin{bmatrix} \frac{(D^4 Vi)}{R_O(D-1)^4} \\ -\frac{(D^3 Vi)}{R_O(D-1)^3} \\ -\frac{(D^3 Vi)}{R_O(D-1)^3} \\ \frac{(D^2 Vi)}{R_O(D-1)^2} \\ Vi \\ -\frac{(DVi)}{D-1} \\ \frac{(D^2 Vi)}{(D-1)^2} \\ \frac{(D^2 Vi)}{(D-1)^2} \end{bmatrix}$$

(1.9)

Calculating *A* and *B*, as defined in [6], yields (1.10) and (1.11).

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{D-1}{L_1} & \frac{D-1}{L_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{D}{L_2} & \frac{D-1}{L_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{D}{L_3} & \frac{D-1}{L_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{D}{L_4} & \frac{D}{L_4} & \frac{D-1}{L_4} - \frac{D}{L_4} \\ -\frac{D-1}{C_1} & -\frac{D}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{D-1}{C_2} & -\frac{D-1}{C_2} & -\frac{D}{C_2} & -\frac{D}{C_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{D-1}{C_3} & -\frac{D}{C_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{D}{C_4} - \frac{D-1}{C_4} & 0 & 0 & 0 & \frac{D-1}{C_4 R_o} - \frac{D}{C_4 R_o} \end{bmatrix}$$

(1.10)

$$B = \begin{bmatrix} \frac{D}{L_1} - \frac{D-1}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1.11)

The C will be composed of just the variable we want as output, from the X vector. It is shown in (1.12)

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(1.12)

The I matrix is the indentity matrix with size equal to the matrix A, being 8 x 8.

With all matrices calculated, we apply (1.2) using *MATLAB* to find the transfer function of the converter. Defining $T_P(s)$ as the transfer function, in (1.13).

$$T_p(s) = \frac{N(s)}{D(s)}$$

(1.13)

With *N(s)* and *D(s)* representing the numerator and denominator of the transfer function. They are shown in (1.14) and (1.15).

$$\begin{aligned}
N(s) = (&DV_i(2R_o + 20D^2R_o - 20DR_o + 10D^3R_o \\
&- 2D^4R_o - 10D^5R_o - \#5 - \#4 + 3D^3L_2s - 2D^4L_1s \\
&+ 3D^3L_3s - 3D^4L_2s + 2D^5L_1s - 3D^4L_3s + D^5L_2s + D^5L_3s + 2C_1L_1R_os^2 + 2C_1L_2R_os^2 \\
&+ \#7 + 2C_3L_3R_os^2 - 10C_1DL_1R_os^2 - 10C_1DL_2R_os^2 \\
&- 5C_2DL_2Ros^2 - 8C_3DL_3R_os^2 - \#16 - \#15 - \#14 \\
&+ 3C_1D^3L_1L_3s^3 + C_1D^4L_1L_2s^3 + 3C_1D^3L_2L_3s^3 - 3C_1D^4L_1L_3s^3 \\
&- \#13 - 3C1D^4L_2L_3s^3 + C_1D^5L_1L_3s^3 + 3C_2D^3L_2L_3s^3 - \#12 - \#11 + C1D^5L2L3s^3 \\
&- 3C_2D^4L_2L_3s^3 + C_2D^5L_1L_3s^3 + 3C_3D^3L_2L_3s^3 - 2C_3D^4L_1L_3s^3 \\
&+ C_2D^5L_2L_3s^3 - 3C_3D^4L_2L_3s^3 + C_3D^5L_1L_3s^3 + C_3D^5L_2L_3s^3 \\
&+ 20C_1D^2L_1R_os^2 + 20C_1D^2L_2R_os^2 - 20C_1D^3L_1R_os^2 + \#10 - 20C_1D^3L_2R_os^2 \\
&+ 10C_1D^4L_1R_os^2 + 10C_2D^2L_2R_os^2 - 3C_2D^3L_1R_os^2 + 10C_1D^4L_2R_os^2 \\
&- 2C_1D^5L_1R_os^2 - 10C_2D^3L_2R_os^2 + 3C_2D^4L_1R_os^2 - 2C_1D^5L_2R_os^2 \\
&+ 5C_2D^4L_2R_os^2 - C_2D^5L_1R_os^2 + 12C_3D^2L_3R_os^2 - C_2D^5L_2R_os^2 \\
&- 8C_3D^3L_3R_os^2 + 2C_3D^4L_3R_os^2 + \#9 + 2C_1C_3L_1L_3R_os^4 + 2C_1C_3L_2L_3R_os^4 \\
&+ \#8 - \#3 + C_1C_2D^3L_1L_2L_3s^5 - \#2 + 3C_1C_2D^2L_1L_2R_os^4 - C_1C_2D^3L_1L_2R_os^4 \\
&+ 12C_1C_3D^2L_1L_3R_os^4 + 12C_1C_3D^2L_2L_3R_os^4 - 8C_1C_3D^3L_1L_3R_os^4 \\
&+ \#1 - 8C_1C_3D^3L_2L_3R_os^4 + 2C_1C_3D^4L_1L_3R_os^4 + 6C_2C_3D^2L_2L_3R_os^4 \\
&- 2C_2C_3D^3L_1L_3R_os^4 + 2C_1C_3D^4L_2L_3R_os^4 - 4C_2C_3D^3L_2L_3R_os^4 \\
&+ C_2C_3D^4L_1L_3R_os^4 + C_2C_3D^4L_2L_3R_os^4 - 3C_1C_2DL_1L_2R_os^4 - 8C_1C_3DL_1L_3R_os^4 \\
&- 8C_1C_3DL_2L_3R_os^4 - 4C_2C_3DL_2L_3R_os^4 + \#6 - 2C_1C_2C_3DL_1L_2L_3R_os^6 \\
&+ C_1C_2C_3D^2L_1L_2L_3R_os^6))
\end{aligned}$$

(1.14)

$$\begin{aligned}
D(s) = (&(D-1)^4(R_o + L_4s + 6D^2R_o - 4D^3R_o + D^4R_o - 4DR_o - 4DL_4s \\
&+ \#5 + \#4 - 2D^3L_2s + D^4L_1s + 6D^2L_4s - 2D^3L_3s + D^4L_2s - 4D^3L_4s + D^4L_3s + D^4L_4s \\
&+ C_1L_1L_4s^3 + C_1L_2L_4s^3 + C_2L_2L_4s^3 + C_3L_3L_4s^3 + C_1L_1R_os^2 + C_1L_2R_os^2 + \#7 \\
&+ C_3L_3R_os^2 + C_4L_4R_os^2 - 4C_1DL_1L_4s^3 - 4C_1DL_2L_4s^3 - 4C_2DL_2L_4s^3 - 2C_3DL_3L_4s^3 \\
&- 4C_1DL_1R_os^2 - 4C_1DL_2R_os^2 - 4C_2DL_2R_os^2 - 2C_3DL_3R_os^2 - 4C_4DL_4R_os^2 + \#16 + \#15 \\
&+ 6C_1D^2L_1L_4s^3 + 14 - 2C_1D^3L_1L_3s^3 + 6C_1D^2L_2L_4s^3 - 4C_1D^3L_1L_4s^3 - 2C_1D^3L_2L_3s^3 \\
&+ C_1D^4L_1L_3s^3 + C_2D^2L_1L_4s^3 + \#13 - 4C_1D^3L_2L_4s^3 + C_1D^4L_1L_4s^3 + C_1D^4L_2L_3s^3 \\
&+ 6C_2D^2L_2L_4s^3 - 2C_2D^3L_1L_4s^3 - 2C_2D^3L_2L_3s^3 + \#12 + \#11 \\
&+ C_1D^4L_2L_4s^3 - 4C_2D^3L_2L_4s^3 + C_2D^4L_1L_4s^3 + C_2D^4L_2L_3s^3 + C_3D^2L_2L_4s^3 \\
&- 2C_3D^3L_2L_3s^3 + C_3D^4L_1L_3s^3 + C_2D^4L_2L_4s^3 + C_3D^2L_3L_4s^3 - 2C_3D^3L_2L_4s^3 \\
&+ C_3D^4L_1L_4s^3 + C_3D^4L_2L_3s^3 + C_3D^4L_2L_4s^3 + 6C_1D^2L_1R_os^2 + 6C_1D^2L_2R_os^2 \\
&- 4C_1D^3L_1Ros^2 + \#10 - 4C_1D^3L_2R_os^2 + C_1D^4L_1R_os^2 + 6C_2D^2L_2R_os^2 - 2C_2D^3L_1R_os_2 \\
&+ C_1D^4L_2R_os^2 - 4C_2D^3L_2R_os^2 + C_2D^4L_1R_os^2 + C_3D^2L_2R_os^2 + C_2D^4L_2R_os^2 \\
&+ C_1D^4L_2R_os^2 - 4C_2D^3L_2R_os^2 + C_2D^4L_1R_os^2 + C_3D^2L_2R_os^2 + C_2D^4L_2R_os^2 \\
&+ C_3D^2L_3R_os^2 - 2C_3D^3L_2R_os^2 + C_3D^4L_1R_os^2 + C_4D^2L_2R_os^2 + C_3D^4L_2R_os^2 \\
&+ C_4D^2L_3R_os^2 - 2C_4D^3L_2R_os^2 + C_4D^4L_1R_os^2 + 6C_4D^2L_4R_os^2 - 2C_4D^3L_3R_os^2 \\
&+ C_4D^4L_2R_os^2 - 4C_4D^3L_4R_os^2 + C_4D^4L_3R_os^2 + C_4D^4L_4R_os^2 + C_1C_2L_1L_2L_4s^5 \\
&+ C_1C_3L_1L_3L_4s^5 + C_1C_3L_2L_3L_4s^5 + C_2C_3L_2L_3L_4s^5 + \#9 + C_1C_3L_1L_3R_os^4 \\
&+ C_1C_3L_2L_3R_os^4 + C_1C_4L_1L_4R_os^4 + \#8 + C_1C_4L_2L_4R_os^4 + C_2C_4L_2L_4R_os^4 \\
&+ C_3C_4L_3L_4R_os^4 + \#3 + C_1C_2D^2L_1L_2L_4s^5 + \#2 + C_1C_3D^2L_1L_2L_4s^5 + C_1C_3D^2L_1L_3L_4s^5 \\
&+ C_1C_3D^2L_2L_3L_4s^5 + C_2C_3D^2L_1L_3L_4s^5 + C_2C_3D^2L_2L_3L_4s^5 + C_1C_2D^2L_1L_2R_os^4 \\
&+ C_1C_3D^2L_1L_2R_os^4 + C_1C_3D^2L_1L_3R_os^4 + C_1C_4D^2L_1L_2R_os^4 + C_1C_3D^2L_2L_3R_os^4 \\
&+ C_1C_4D^2L_1L_3R_os^4 + \#1 + 6C_1C_4D^2L_1L_4R_os^4 + C_1C_4D^2L_2L_3R_os^4 - 2C_1C_4D^3L_1L_3R_os^4 \\
&+ C_2C_3D^2L_2L_3R_os^4 + 6C_1C_4D^2L_2L_4R_os^4 - 4C_1C_4D^3L_1L_4R_os^4 - 2C_1C_4D^3L_2L_3R_os^4 \\
&+ C_1C_4D^4L_1L_3R_os^4 + C_2C_4D^2L_1L_4R_os^4 + C_2C_4D^2L_2L_3R_os^4 - 4C_1C_4D^3L_2L_4R_os^4 \\
&+ C_1C_4D^4L_1L_4R_os^4 + C_1C_4D^4L_2L_3R_os^4 + 6C_2C_4D^2L_2L_4R_os^4 - 2C_2C_4D^3L_1L_4R_os^4 \\
&- 2C_2C_4D^3L_2L_3R_os^4 + C_2C_4D^4L_1L_3R_os^4 + C_3C_4D^2L_2L_3R_os^4 + C_1C_4D^4L_2L_4R_os^4 \\
&- 4C_2C_4D^3L_2L_4R_os^4 + C_2C_4D^4L_1L_4R_os^4 + C_2C_4D^4L_2L_3R_os^4 + C_3C_4D^2L_2L_4R_os^4 \\
&- 2C_3C_4D^3L_2L_3R_os^4 + C_3C_4D^4L_1L_3R_os^4 + C_2C_4D^4L_2L_4R_os^4 + C_3C_4D^2L_3L_4R_os^4 \\
&- 2C_3C_4D^3L_2L_4R_os^4 + C_3C_4D^4L_1L_4R_os^4 + C_3C_4D^4L_2L_3R_os^4 + C_3C_4D^4L_2L_4R_os^4 \\
&- 2C_1C_2DL_1L_2L_4s^5 - 2C_1C_3DL_1L_3L_4s^5 - 2C_1C_3DL_2L_3L_4s^5 - 2C_2C_3DL_2L_3L_4s^5 \\
&- 2C_1C_2DL_1L_2R_os^4 - 2C_1C_3DL_1L_3R_os^4 - 2C_1C_3DL_2L_3R_os^4 - 4C_1C_4DL_1L_4R_os^4 \\
&- 2C_2C_3DL_2L_3R_os^4 - 4C_1C_4DL_2L_4R_os^4 - 4C_2C_4DL_2L_4R_os^4 - 2C_3C_4DL_3L_4R_os^4 \\
&+ C_1C_2C_3L_1L_2L_3L_4s^7 + \#6 + C_1C_2C_4L_1L_2L_4R_os^6 + C_1C_3C_4L_1L_3L_4R_os^6 \\
&+ C_1C_3C_4L_2L_3L_4R_os^6 + C_2C_3C_4L_2L_3L_4R_os^6 - 2C_1C_2C_4DL_1L_2L_4R_os^6 \\
&- 2C_1C_3C_4DL_1L_3L_4R_os^6 - 2C_1C_3C_4DL_2L_3L_4R_os^6 - 2C_2C_3C_4DL_2L_3L_4R_os^6 \\
&+ C_1C_2C_4D^2L_1L_2L_3R_os^6 + C_1C_2C_4D^2L_1L_2L_4R_os^6 + C_1C_3C_4D^2L_1L_2L_3R_os^6 \\
&+ C_1C_3C_4D^2L_1L_2L_4R_os^6 + C_1C_3C_4D^2L_1L_3L_4R_os^6 + C_1C_3C_4D^2L_2L_3L_4R_os^6 \\
&+ C_2C_3C_4D^2L_1L_3L_4R_os^6 + C_2C_3C_4D^2L_2L_3L_4R_os^6 + C_1C_2C_3C_4L_1L_2L_3L_4R_os^8))
\end{aligned}$$

(1.15)

With additional values (1.16).

$$\begin{aligned}
\#1 &= C_2C_3D^2L_1L_3R_os^4 \\
\#2 &= C_1C_3D^2L_1L_2L_3s^5 \\
\#3 &= C_1C_2D^2L_1L_2L_3s^5 \\
\#4 &= D^2L_3s \\
\#5 &= D^2L_2s \\
\#6 &= C_1C_2C_3L_1L_2L_3R_os^6 \\
\#7 &= C_2L_2R_os^2 \\
\#8 &= C_2C_3L_2L_3R_os^4 \\
\#9 &= C_1C_2L_1L_2R_os^4 \\
\#10 &= C_2D^2L_1R_os^2 \\
\#11 &= C_3D^2L_2L_3s^3 \\
\#12 &= C_2D^4L_1L_3s^3 \\
\#13 &= C_2D^2L_2L_3s^3 \\
\#14 &= C_1D^2L_2L_3s^3 \\
\#15 &= C_1D^2L_1L_3s^3 \\
\#16 &= C_1D^2L_1L_2s^3
\end{aligned}$$

(1.16)

3. Power stage design

This section will be dedicated to comparing the responses of the equation acquired by the averaged space-state modeling with the simulation, aiming at validating the theoretical analysis. By substituting the component values calculated in this section in (1.13), we will acquire a polynomial that will describe the converter behaviour.

Tab. 1 shows the project parameters for the power stage.

Parameter	Value
Input Voltage	V_i=12 V
Output voltage	V_O=127 V
Duty Cycle	D = 0.7648
Load resistance	R_O = 161.29 Ω
Output power	P_O = 100 W
Switching Frequency	f_s = 30 kHz
Peak-to-Peak current ripple on inductors	ΔI_{L1} =10% I_{L1} *(md.)*
	ΔI_{L2} = 20% I_{L1} *(md.)*
	ΔI_{L3} = 20% I_{L1} *(md.)*
	ΔI_{L4} = 20% I_{L1} *(md.)*
Peak-to-Peak voltage ripple on inductors	ΔV_{C1} = 5% V_{C1}
	ΔV_{C2} = 5% V_{C2}
	ΔV_{C3} = 5% V_{C3}
	ΔV_{C4} = 1% V_O

By using the equations derived on [5], we calculate the components needed for the converter's power stage. They are presented in (1.17) to (1.25).

$$I_o = \frac{P_o}{V_o} = 0.787\text{A}$$

(1.17)

$$L_1 = \frac{DV_i}{f_s \Delta I_{L1}} = 367.427\mu\text{H}$$

(1.18)

$$L_2 = \frac{DV_i}{f_s \Delta I_{L2}} = 597.5\mu\text{H}$$

(1.19)

$$L_3 = \frac{D^2 V_i}{(1-D) f_s \Delta I_{L3}} = 1.943\text{mH}$$

(1.20)

$$L_4 = \frac{D^2 V_i}{(1-D) f_s \Delta I_{L4}} = 6.32\text{mH}$$

(1.21)

$$C_1 = \frac{D^2 I_o}{(1-D) f_s \Delta V_{C1}} = 108.788\mu\text{F}$$

(1.22)

$$C_2 = \frac{D^2 I_o}{(1-D) f_s \Delta V_{C2}} = 33.46\mu\text{F}$$

(1.23)

$$C_3 = \frac{(1-D) I_o}{f_s \Delta V_{C3}} = 972.162\text{nF}$$

(1.24)

$$C_4 = \frac{D^2 Vi}{8 f_s^2 L_4 \Delta V_{C4}} = 121.459\text{nF}$$

(1.25)

3.1 Determining the transfer function $V_o(s)/d(s)$

After determining the component values of the power stage, we solve equation (1.13) to find the polynomials. The numerical $T_P(s)$ is shown in (1.26).

$$\frac{2.163 \cdot 10^{11} s^6 - 3.857 \cdot 10^{15} s^5 + 3.231 \cdot 10^{19} s^4 - 1.586 \cdot 10^{23} s^3 + 7.005 \cdot 10^{26} s^2 - 1.261 \cdot 10^{30} s + 3.725 \cdot 10^{33}}{s^8 + 5.105 \cdot 10^4 s^7 + 1.457 \cdot 10^9 s^6 + 7.856 \cdot 10^{12} s^5 + 7.669 \cdot 10^{16} s^4 + 1.967 \cdot 10^{20} s^3 + 9.084 \cdot 10^{23} s^2 + 1.216 \cdot 10^{27} s + 2.637 \cdot 10^{30}}$$

(1.26)

3.2 Circuit Simulation

With equation (1.26) at hand, a disturbance signal will be applied with small variations, and added to the median output voltage. That way, when both waveforms are superimposed (converter simulation and transfer funtion, both time responses) it will become evident that the equation response it the median value of the converter response.

PSIM will be used for the circuit and transfer function simulations. The circuits diagram is shown in **Fig. 2**.

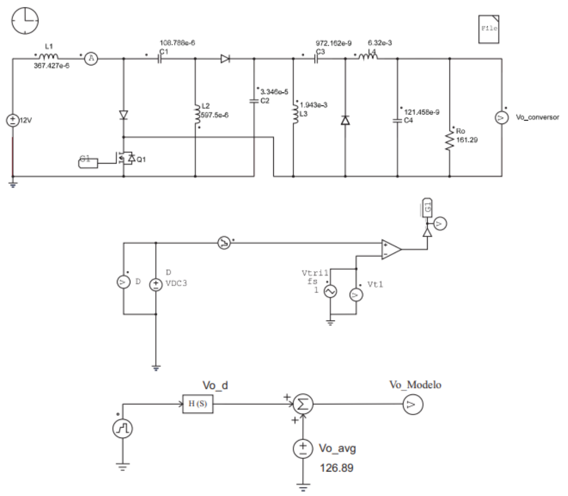

Fig. 2. Simulated circuits on *PSIM.* From top to bottom: Power circuit, PWM generator and Transfer function simulator.

The system described by **Fig. 2** simulates the real behaviour of the circuit, in continous conduction mode, without losses. The duty cycle will be disturbed by +0.005 *V*, -0.01 *V* and +0.01 *V* in 30 m*s*, 55 m*s* and 80 m*s*, both in the PWM generator and the *H(s)* block.

4. Results

Fig. 3 shows the two waveforms, superimposed.

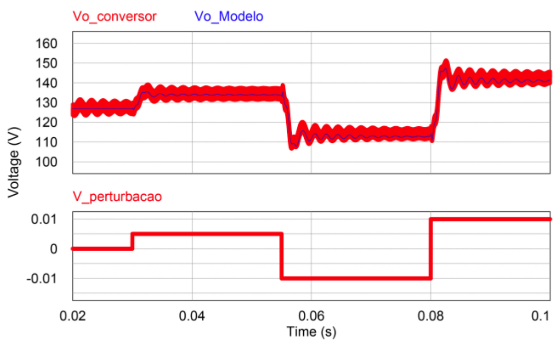

Fig. 3. Output voltage response obtained by the averaged model and by the converter faced with small disturbances applied to the duty cycle.

The first graph shows the converter response in red, and the model response in blue. We can clearly observe that the transfer function illustrates the median value of the converter response, validating the model.

The second graph shows the disturbance applied to the duty cycle.

5. Conclusion

This paper presented the small signal modeling for the SEPIC-Zeta converter, operating in continuous conduction mode. Starting with an mathematical approach, the averaged space-state equations were derived. Although complex, with long equation and the need of symbolic computation in third party softwares, it is verified that the techinique yields the correct transfer function of the aforementioned converter, faced with disturbances.

Verifying the results, we can confirm that the technique proposed by Middlebrook and Ćuk in 1976 [4] is adequate to represent the SEPIC-Zeta converter as a voltage lifter, in continuous conduction mode.

6. Acknowledgement

The author would like to thank the Vice-Rectory of Research and Post Graduation (PROPE), and to the Nation Counsil of Cientific Research (CNPq) for the financial aid.

7. References

[1] T.-F. Wu and Y.-K. Chen, Origin of Power Converters: Decoding, Synthesizing, and Modeling. John Wiley & Sons, 2020.

[2] D. A. Tavares, “Concepção de conversores cc-cc não isolados integrados com ampla taxa de conversão.,” Dissertação de mestrado - UFSJ, São João del-Rei - MG, 2012.

[3] J. W. Kolar, U. Drofenik, J. Biela, M. L. Heldwein, H. Ertl, T. Friedli, and S. D. Round, “Pwm converter power density barriers,” in 2007 Power Conversion Conference - Nagoya, pp. P–9–P–29, 2007.

[4] R. D. Middlebrook and S. Cuk, “A general unified approach to modelling switchingconverter power stages,” in 1976 IEEE power electronics specialists conference, pp. 18–34, IEEE, 1976.

[5] K. M. M. Damasceno, M. M. G. Costa, C. M. do Amaral, E. R. Ribeiro, and F. L. Tofoli, “Conversor cc-cc não isolado sepic-zeta,” ANAIS DO XXIV CONGRESSO BRASILEIRO DE AUTOMÁTICA – CBA 2022, vol. 1, 2022.

[6] P. Chetty, “Current injected equivalent circuit approach to modeling switching dc-dc converters,” IEEE Transactions on Aerospace and Electronic Systems, no. 6, pp. 802–808, 1981.

[7] Y. Lee, “A systematic and unified approach to modeling switches in switch-mode power supplies,” IEEE Transactions on Industrial Electronics, no. 4, pp. 445–448, 1985.

[8] R. W. Erickson and D. Maksimovic, Fundamentals of power electronics. Springer Science & Business Media, 2007.